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MODELING THE EFFECT OF FRACTURE ORIENTATION ON PORUS MEDIA ANISOTROPY USING THE T-MATRIX METHOD

Fracture orientation is crucial in porous media anisotropy. In the current study we investigate the role of fracture orientation based on a novel Rock Physics method - the T-matrix approach. This method allows us to study the fracture porosity, shape and orientation together with the minerals volumetric and morphological characteristics.

In the first part of this paper, we assume that the considered system consists of a set of aligned, penny-shaped fractures. To include the fracture's spatial alignment, we use the Euler angles. The sensitivity of the stiffness components on the main diagonal of the stiffness matrix along the extended Thompson parameters to the fracture orientation change was investigated.

Based on the obtained results we concluded that the effect of fracture orientation on the anisotropy of the system can be explained by introducing a hypothetical qualitative parameter called the fracture projection cross-sectional area on the three perpendicular planes (x1, x2 and x3). We developed three basic rules to describe how fracture alignment controls this hypothetical parameter.

Keywords: *anisotropy, T-matrix method, Euler angles, stiffness matrix, fracture orientation.*

Introduction

One of the main goals of seismic exploration is the detection and characterization of fractures. The presence of cracks and their characteristics significantly control the physical properties of the porous medium, such as elastic moduli, permeability, thermal conductivity, electrical resistance, etc. Fracture characterization means assessing the shape, orientation, and porosity of fractures in a porous system. Knowledge of the stiffness matrix components and the anisotropy system determines the structural arrangement of rock components such as the shape and orientation of minerals, pores and cracks. Fracture characterization is simpler at the core scale than at larger scales such as well logging and seismic. Several theoretical methods can be found for evaluating the effect of fractures on the stiffness matrix in [Budiansky, O'Connell, 1976; Hudson, 1980; Schoenberg, Sayers, 1995; Hudson, 2008]. Typically, fractures are modeled as a penny-shaped ellipsoid with a low aspect ratio. Kachanov M. et al. used the non-interacting Eshelby approximation (NIA) and derived expressions to calculate the effect of arbitrary cracks on the effective elasticity [Kachanov, Tsukrov, Shafiro, 1994]. A more detailed discussion of the non-ellipsoidal shape can be found in [Kachanov, Sevostianov, 2018]. The impact of fracture orientation can be simply included in the expressions obtained in the above studies using Euler angles. These theoretical methods can be used to correlate the seismic signature with the properties of fracture systems. Bakulin A. et al. [Bakulin, Grechka, Tsvankin, 2000] used the methods of Sayer C.M and Hudson J.A. [Schoenberg, Sayers, 1995; Hudson, 1980] to theoretically study the effect of fracture orientation on the seismic response. They found linearized expressions between the known Thompson parameters and the Vs/Vp ratio (Vs and Vp are the velocities of the fast transverse and longitudinal waves, respectively). Azimuth analysis of the AVO P-wave was used to detect azimuthal anisotropy and fracture orientation. [Zheng, Todorovic-Marinic, Larson, 2004] proved that the fracture orientation detected from AVO data is not unique and is sensitive to the phase of the seismic data and the type of geological interfaces. In their article, the simulated environment is considered VTI or HTI. Based on the assumption, the direction of the calculated NMO velocity depends on the parameter δv (negative for the HTI medium). These results are consistent with those reported in [Bakulin, Grechka, Tsvankin, 2000; Fang et al., 2014] presented the fracture transfer function (FTF), which represents the ability of the fracture layer to generate scattered waves, and showed that the response of the fracture system can be obtained by calculating the FTF. The direction of the crack can be determined by determining the maximum value of the average FTF.

In this paper, the T-matrix method [Jakobsen, Hudson, Johansen, 2003] was used to estimate the effective stiffness matrix of a fractured medium containing cracks of different spatial orientations. We will attempt to describe the reasons for the effect of fracture orientation on seismic anisotropy using a simple petroleastic model, built using the mentioned T-matrix approach.

Methods and modeling description

To model the fractured system, we use the T-matrix approach [Jakobsen, Hudson, Johansen, 2003]. The equation for the optical generalized potential approximation of T-matrix is as follow:

$$\mathbb{C}^{*} = \mathbb{C}^{(0)} + \langle \mathbb{t} \rangle - \langle \mathbb{t} \rangle : \mathbb{G} : \langle \mathbb{t} \rangle \quad (1).$$

Where,

$$\mathbb{t}^{(i)} = \left(\mathbb{I} - \delta\mathbb{C}^{(i)} : \mathbb{G}^{(i)} \right)^{-1} : \delta\mathbb{C}^{(i)} \quad (2).$$

$\mathbb{I}$ is a unit fourth rank tensor. $\mathbb{C}^{*}$, $\mathbb{C}^{(0)}$ and $\delta\mathbb{C}^{(i)}$ are effective stiffness tensor, embedding matrix stiffness tensor and fluctuation of i’th component stiffness tensor relative to the embedding matrix stiffness tensor respectively. $\mathbb{G}^{(i)}$, is the second derivative of the Green’s functions and can be computed as follow:

$$G_{kmln} = \tilde{a}_{k)(l,n)(m} \quad (3)$$

$$\tilde{a}_{k)(l,n)(m} = \frac{1}{4}(a_{klnm} + a_{mlnk} + a_{knlm} + a_{mnlk}) \quad (4)$$

$$a_{kmln} = \frac{1}{4\pi}\int_{0}^{2\pi}\int_{0}^{\pi} n_{mn}\Lambda_{kl}^{-1} sin\theta d\theta d\phi \quad (5)$$

$$\Lambda_{kl} = C^{(0)}_{kmln} n_{mn} \tag{6}$$

$$n_{mn} = n_n n_m \tag{7}$$

$$n_1 = \frac{1}{a_1} sin\theta \cdot cos\phi \tag{8}$$

$$n_2 = \frac{1}{a_2} sin\theta \cdot sin\phi \tag{9}$$

$$n_3 = \frac{1}{a_3} cos\theta \qquad (10).$$

Where a1, a2 and a3 are semi-axis of the ellipsoidal inclusions. n is the cosine vector. Triangle brackets in equation (1) denote volume averaging over a representative volume [Shermergor, 1977]:

$$\langle \mathbb{X} \rangle = \frac{v^i}{8\pi^2} \int_{\alpha_1^{LB}}^{\alpha_1^{UB}} \int_{\alpha_2^{LB}}^{\alpha_2^{UB}} \int_0^{\pi} \int_0^{2\pi} \int_0^{2\pi} \mathbb{X}(\alpha_1, \alpha_2, \theta, \phi, \psi)\, P(\alpha_1, \alpha_2, \theta, \phi, \psi) sin\theta d\theta d\phi d\psi d\alpha_1 d\alpha_2 \tag{11}$$

Where α_1 and α_2 are the two aspect ratios of a three-dimensional ellipsoid. P is the multivariate probability distribution function. $\theta, \phi, and\ \psi$ are the Euler angles. Details of using Euler angle and computation of rotation matrix is presented in Appendix II. vi is the volume fraction of i'th inclusion. UB and LB denote the lower and upper bounds of the aspect ratios. $\mathbb{X}$ is an arbitrary tensor in the triangle bracket.

In the present study, inclusion shape effects on the stiffness tensor components. is ignored and will be discussed elsewhere. We assume that the spatial orientation of fractures is a function of θ. Therefore, equation (11) is simplified as follow:

$$\langle \mathbb{X} \rangle = \frac{v^i}{8\pi^2} \int_0^{2\pi} \mathbb{X}(\theta)\, P(\theta) sin\theta d\theta \qquad (12).$$

The obtained results for the above case can be simply extended to more general cases of spatial orientation - $P(\theta, \phi, \psi)$.

The modeled fractured medium is a mixture of 52% calcite, 47.5% dolomite and 0.5% quartz with total porosity of 0.5%. We assume that calcite, dolomite, and quartz are in the form of isotropic polycrystals. The bulk moduli of calcite, dolomite and quartz polycrystals are 64.51, 91.76, and 35.94 GPa and their shear moduli are 27.72, 35.92, and 41.77 GPa, respectively. The shape of mineral inclusions is considered to be spherical. The embedding matrix stiffness tensor is obtained by averaging the upper and lower bounds of bulk and shear moduli of the mineral polycrystal mixture, computed using Hashin-Shtrikhman method [Hashin,S htrikman, 1963]. Then the penny-shape fractures with the aspect ratio α=10-3 are inserted in the homogenized mineral mixture. The obtained medium is then homogenized using the T-matrix method (see equation (1)) considering various spatial orientation of fractures (equation (12)). The constructed model was considered dry, i.e., the saturating fluid is air in standard condition (60° F and 14.7 psi). Air behavior at the standard condition can be approximated by ideal gas assumption. As a result, the bulk modulus can be easily computed by the following expression:

$$k = P \qquad (13).$$

Where P is the ideal gas pressure. According to the above expression, the bulk modulus of air is equal to 0.000101GPa.

In the present paper, space is referenced by a rectangular coordinate system x1x2x3, where x3 is chosen as the vertical direction with respect to the Earth's surface. The seismic measurements are assumed to be conducted along x1 and x2 axis. Rotation is characterized by Euler angles – ϕ, θ and ψ and which indicate rotation around x3, x′1, and x_3' axis, respectively.

Results and discussion

We investigated simple orientation of fractures. No statistical distribution function was considered. In this case, we simply change the orientation of fractures in three dimensions. Since the fractures are penny shape, two Euler angles are sufficient to characterize the fractures spatial orientation – θ and ϕ.

Fractures are horizontally aligned in x3 plane for θ=0 and φ=0 values by increasing θ from zero to π/2, and keeping the ϕ values constant fractures are spinning around the x1- axis. Fig. 1 demonstrates how changing θ and φ angles affects the diagonal components of the stiffness matrix. According to the presented results in this figure, sensitivity of stiffness matrix components to fracture orientation angles is noticeable. This means that these angles can be effectively considered as unknown parameters in inverse problem solution.

The x2x3 plane is the compressional wave propagation plane of the c11 component, similarly the x1x3 plane, is the compressional wave propagation plane of the c22 component and the x1x2plane, is the compressional wave propagation plane of the c33 component. «The x2x3 plane is the compressional wave propagation plane of the c11 component».

The main reason behind the observed changes of diagonal components by changing Euler angles is the complicated relation between the stiffness matrix components and the Euler angles, established through the tensor form of the rotation matrix. This relation becomes more intricate and practically incomprehensible when integration is employed on orientation. To solve this problem and make the observed changes for this case and the following cases easier to understand we suggest some conceptual and abstract rules.

Accordingly, we can set the following remark: «The first three diagonal components of the stiffness matrix (i.e., c11, c22, and c33) are functions of cross-sectional area[1] of fracture projection on their respective compressional wave propagation planes».

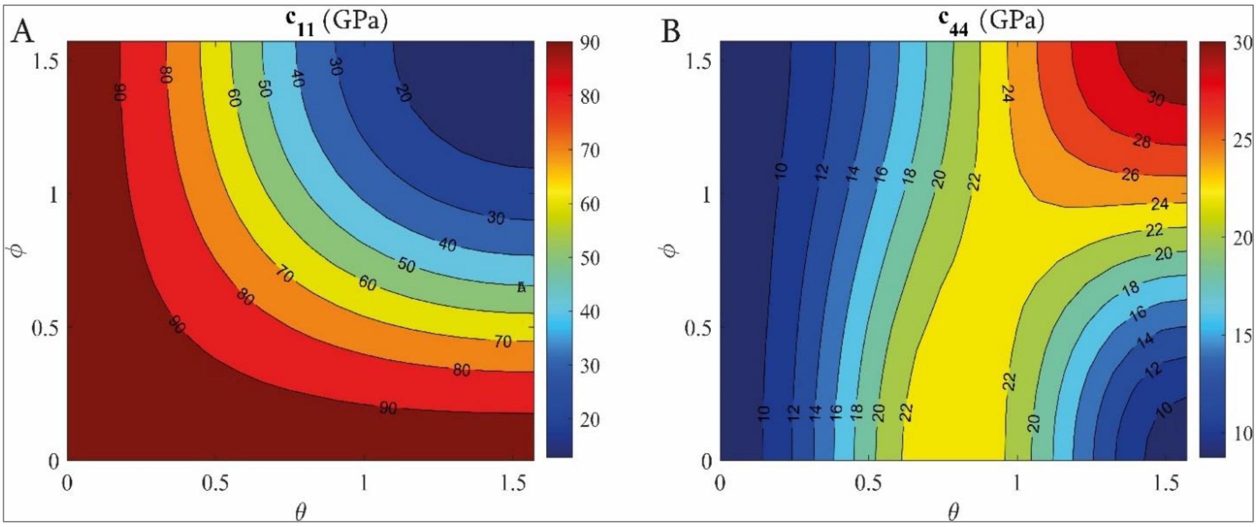

Fig. 1. Variation of the diagonal components of the elasticity tensor versus fracture orientation

According to this remark, the c11, depends on the cross-sectional area of fracture projection on x2x3 plane. Because of the penny form of the considered fractures keeping the θ value equal to zero and changing j values have no effect on the c11 values. However, simultaneous increase of θ and φ angles decreases c11 because of increasing fracture projection cross sectional area in x2x3 plane (Fig. **1**A). Similarly, we can simply explain the effect of change of φ angle on c11 values in Fig. 1A and the effect of change of θ and φ angles on c22, c33, c44, c55 and c66 values (Fig. 1B).

The next issue is to characterize the anisotropy of the constructed modeled media. We have the well-known Thompson's parameters for TI symmetries and Tsvankin's anisotropy parameters for orthorhombic media [Tsvankin, 1997]. However, the anisotropy system of the constructed media in this study can be lower. For systems of higher symmetries than orthorhombic additional anisotropy parameters are introduced which are responsible for rotation of P, S1 and S2 – waves NMO ellipses ($\zeta^{(1,2,3)}$). Further, we follow Grechka's approach to characterize the anisotropy of the studied modeled media [Grechka, 2000]. The parameters ε(1), ε(2), ε(3), g(1), g(2), and g(3) are the measures of anisotropy of compressional and shear waves velocities propagating in x1, x2, and x3 directions, respectively. Since we consider seismic surveys along x1 and x2 directions, we skip the anisotropy parameters related to the azimuth angles from zero to π/2, i.e., $\zeta^{(1,2,3)}$.

Fig. 2 elucidates the effect of fracture orientation on the Grechka's anisotropy parameters. These parameters equal zero in isotropic media. δ is called "strange anisotropy parameter" [Tsvankin,

[1] *The cross-sectional area in this context has a hypothetical rather than a quantitative concept.*

1995]. $\delta(1)$, and $\delta(2)$, characterize the P - waves velocities, propagating in, x3, (both in vertical directions) direction and $\delta(3)$ controls the P - wave propagation in x1 direction. Furthermore $\delta(1)$, $\delta(2)$ and $\delta(3)$ govern shear waves polarized in x1, x2, and x3 planes respectively. They are also indicative of the P-wave velocity at near critical incidence [Tsvankin, 1997].

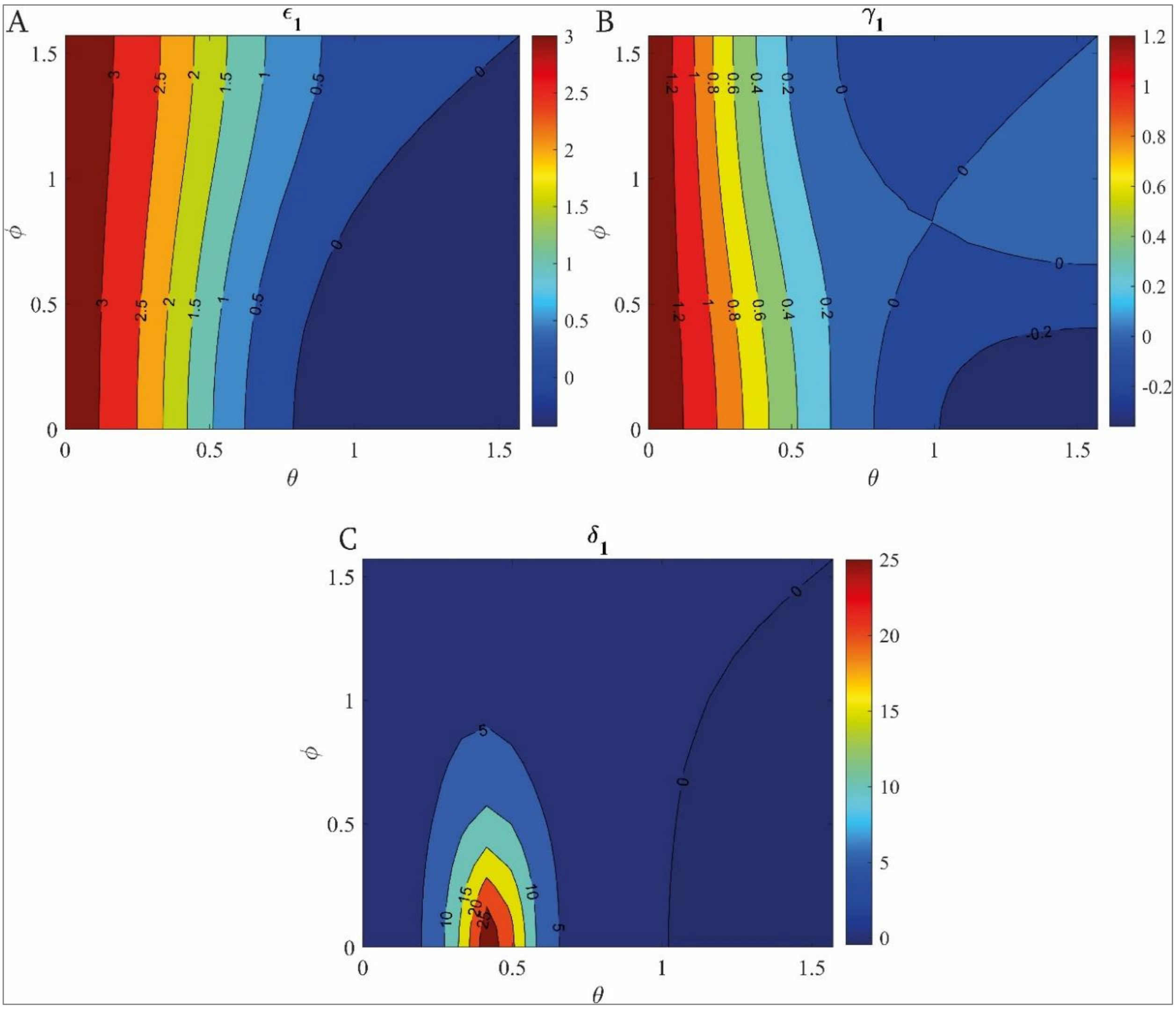

Fig. 2. Changing index parameters of the intensity anisotropy related to fracture orientation

According to Fig. 2, $\varepsilon(1)$ and $\varepsilon(2)$ have positive values when the orientation of the fracture system is close to horizontal (pseudo-vertical transversal isotropy (VTI-symmetries) system), and negative values for horizontal transversal isotropy (HTI-symmetries). Moreover, $\varepsilon(3)$ is negative for all values of ϕ greater than $\pi/4$. The maximum anisotropy of longitudinal waves propagating in the x1 and x2 directions (i.e., the maximum value of $\varepsilon(3)$) is expected for the VTI symmetry system. The sensitivity of the shear wave anisotropy parameter - parameter g is shown in Fig. 2.

The discussed results can be summarized as follows:

– Three main remarks were established to describe the reason behind the effect of orientation and dispersion degree changes on stiffness components.

–Describing fracture orientation with Euler angles, yields to a transverse isotropy symmetry (hexagonal symmetry).

–- In such a system, degree of anisotropy can be high and even contradicts with Thomsen's weak anisotropy assumption.

–- NMO data interpretation is challenging for such a system.

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МОДЕЛИРОВАНИЕ ВЛИЯНИЯ ОРИЕНТАЦИИ ТРЕЩИН НА АНИЗОТРОПИЮ ПОРИСТОЙ СРЕДЫ С ПОМОЩЬЮ МЕТОДА Т-МАТРИЦЫ

Ориентация трещин имеет решающее значение в анизотропии пористой среды. В настоящем исследовании изучена роль ориентации трещины на основе нового метода физики горных пород - Т-матрицы. Этот метод позволяет учитывать пористость, форму и ориентацию трещин наряду с объемными и морфологическими характеристиками минералов. Предположено, что рассматриваемая система состоит из набора выровненных трещин в форме монеты. Для включения пространственного выравнивания трещин используются углы Эйлера. Исследована чувствительность компонентов жесткости на главной диагонали матрицы жесткости по расширенным параметрам Томпсона к изменению ориентации трещины. Основываясь на полученных результатах, сделан вывод о влиянии ориентации трещины на анизотропию системы, что можно объяснить введением гипотетического качественного параметра, называемого площадью поперечного сечения проекции трещины в трех перпендикулярных плоскостях (x_1, x_2, и x_3). Разработано три основных правила для описания того, как выравнивание трещин влияет на этот гипотетический параметр.

Ключевые слова: *анизотропия пористой среды, метод Т-матрицы, углы Эйлера, матрица жесткости, ориентация трещины.*

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